

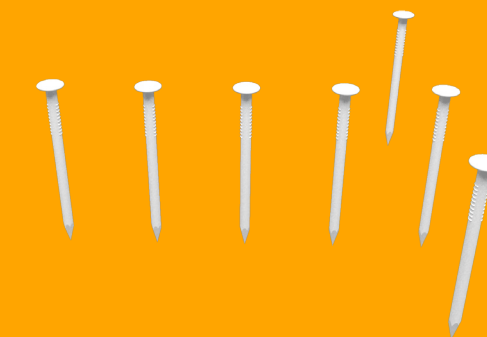
RECENT PAPER DECENT PUZZLE

with DAN RISKIN

NAILS AND STRING

PUZZLE # 027

JANUARY 30, 2016



NAILS AND STRING

There's a board in front of you with a horizontal row of five nails sticking out of it. They are spaced every 2 cm, and are labeled from right to left, 'A, B, C, D, and E'. There is another nail exactly 5 cm above 'E', labeled '1'. As well as a nail exactly 3 cm below 'E', labeled '2'.

You have a piece of string, with loops on the ends, that is exactly 14 cm long.

The Question: How do you loop the string through the nails so that it's pulled taut, and one end is at nail '1', and the other end at nail '2'?

You can touch as many nails as you want, but you can't loop it around the same nail many times to shorten it by making a spool.

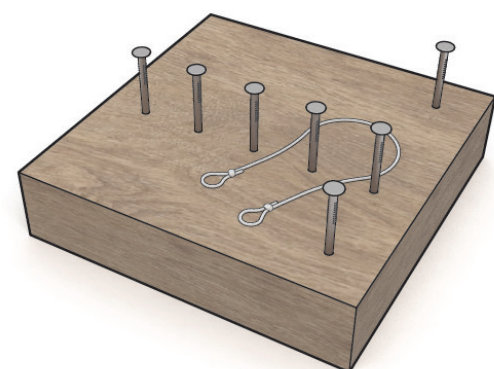


Fig 1: Nails in a board, with a 14 cm long string

How We Can Approach The Problem

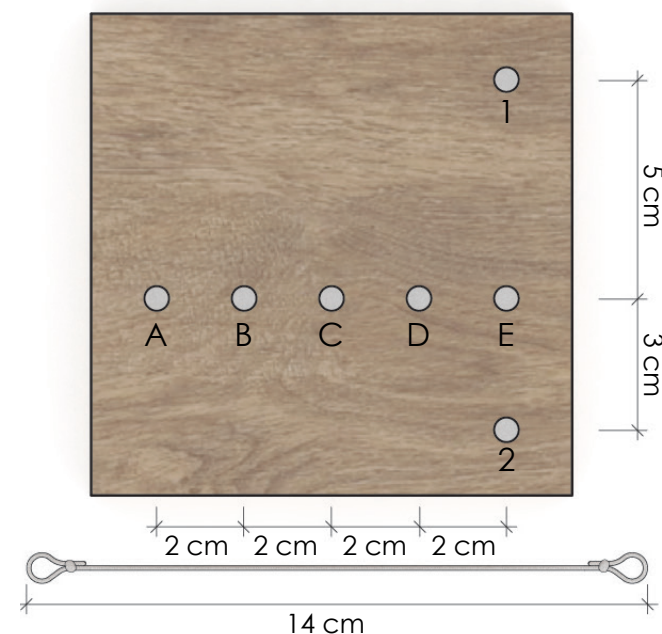


Fig 2: Nails 'A, B, C, D, and E' all 2 cm apart. Nail '1' 5 cm above 'E', and nail '2' 3 cm below 'E'

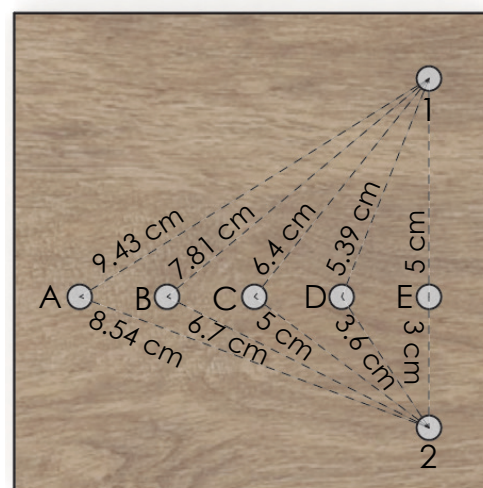
The Puzzle:

The goal is to find a path that starts at nail '1', ends at nail '2', and is exactly 14 cm long.

We know the distance between nails 'A to E', as well as between nail '1, E, and 2'.

We do not know the distance between nail '1' and 'A to D', or between nail '2' and 'A to D'.

Getting More Information



$$\begin{aligned} A1 &= \sqrt{8^2+5^2} = 9.43 \text{ cm} \\ B1 &= \sqrt{6^2+5^2} = 7.81 \text{ cm} \\ C1 &= \sqrt{4^2+5^2} = 6.4 \text{ cm} \\ D1 &= \sqrt{2^2+5^2} = 5.39 \text{ cm} \end{aligned}$$

$$\begin{aligned} A2 &= \sqrt{8^2+3^2} = 8.54 \text{ cm} \\ B2 &= \sqrt{6^2+3^2} = 6.7 \text{ cm} \\ C2 &= \sqrt{4^2+3^2} = 5 \text{ cm} \\ D2 &= \sqrt{2^2+3^2} = 3.6 \text{ cm} \end{aligned}$$

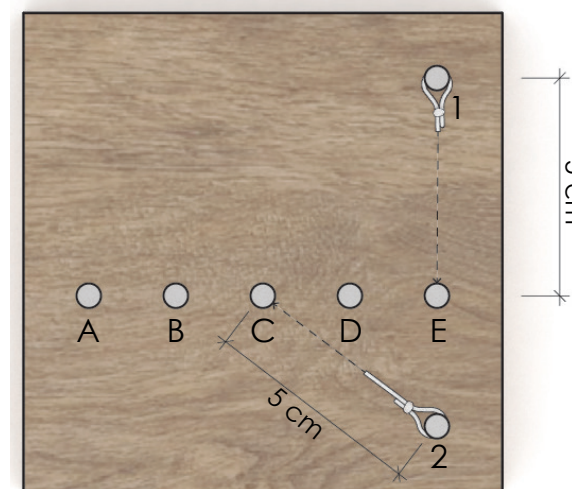
Fig 3: Distance between nails '1, A to D', and '2, A to D'

Distances Between The Nails:

We can tell from the start that the diagonal distances between the nails is going to be important in this problem, since there is no way of making a 14 cm path in an orthogonal route.

Using the Pythagoras Theorem, we can find the missing dimensions.

The Start And End Of The String



PATH

Start 1 to E = 5 cm

End C to 2 = 5 cm

Total = 10 cm

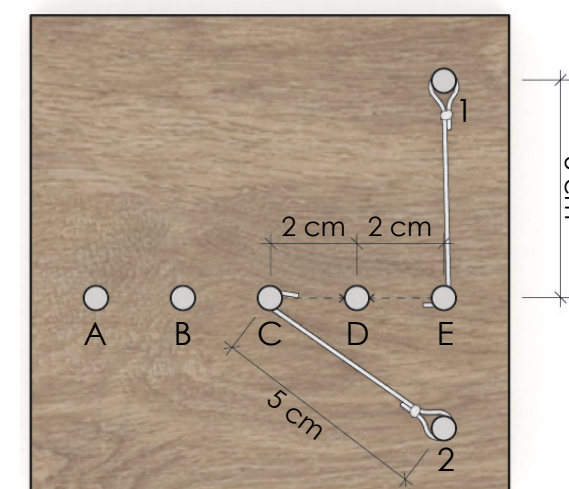
Fig 4: Starting the path at nail '1', and ending the path at nail '2'

Starting The Path:

By looking at the diagonal distances between the nails, it becomes clear that none of the fractions of centimeters add up to a whole number.

The only routes that are not fractions are '1 to E', and 'C to 2'. These need to be the starting and ending routes of the 14 cm path.

Connecting The Middle



PATH

Start 1 to E = 5 cm

Connect E to D = 2 cm

Connect D to C = 2 cm

End C to 2 = 5 cm

Total = 14 cm

Fig 5: Connecting the two ends

The 14 cm Path:

Now, the two ends need to be connected. By continuing the path from nail 'E to D', and then 'D to C' we complete a path of exactly 14 cm.

Solution

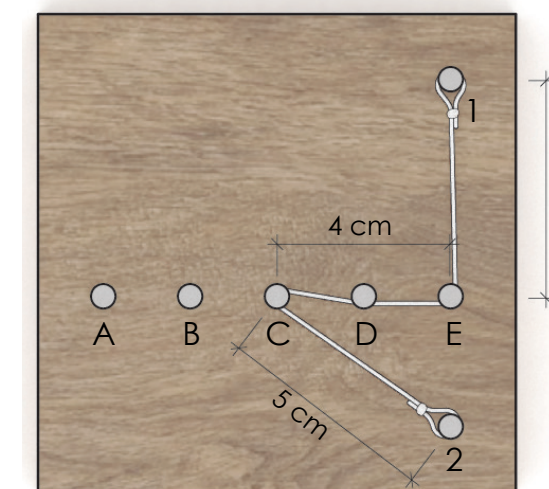


Fig. 6: The solution, in plan, of the 14 cm long path

Path: '1 to E', 'E to D', 'D to C', and 'C to 2'

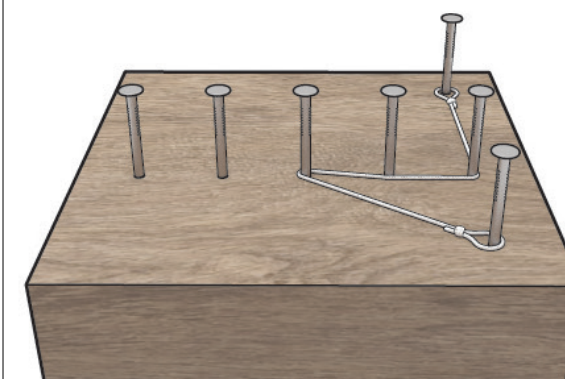


Fig. 7: The solution connecting nail '1' to nail '2' with a path of exactly 14 cm