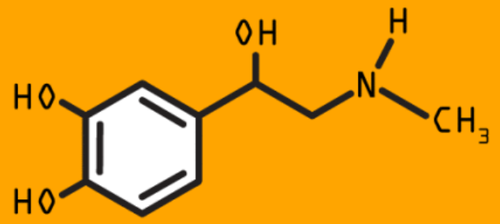

RECENT PAPER DECENT PUZZLE

with DAN RISKIN

ROLLING A SIX

PUZZLE # 029

FEBRUARY 13, 2017

The Question

If you roll a fair six-sided die, the odds of rolling a six on the first roll are 1 in 6, or 16.667%.

On average, how many rolls does it take for you to roll a six?

Fig 1: Rolling a six

Sample Dice Sequence

Roll 1	Roll 2	Roll 3	Roll 4	Roll 5	Roll 6	Roll 7
6 = 16.7%	6 = 16.7%	6 = 16.7%	6 = 16.7%	6 = 16.7%	6 = 16.7%	6 = 16.7%
$(5/6)^1$ = 83.3%	$(5/6)^2$ = 69.4%	$(5/6)^3$ = 57.9%	$(5/6)^4$ = 48.2%	$(5/6)^5$ = 40.2%	$(5/6)^6$ = 33.5%	$(5/6)^7$ = 27.9%

Fig 2: Sample sequence of dice rolls until a six is rolled

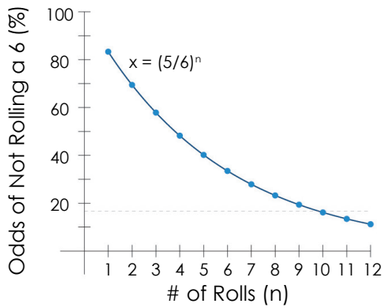

Fig 3: Odds of not rolling a 6

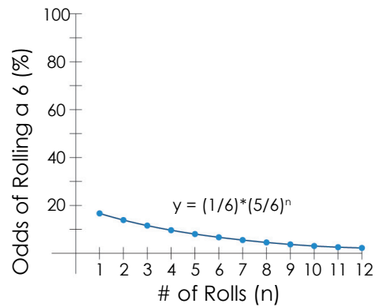

Fig 4: Odds of getting a 6 on each roll

The Odds Of A 6 On Each Roll

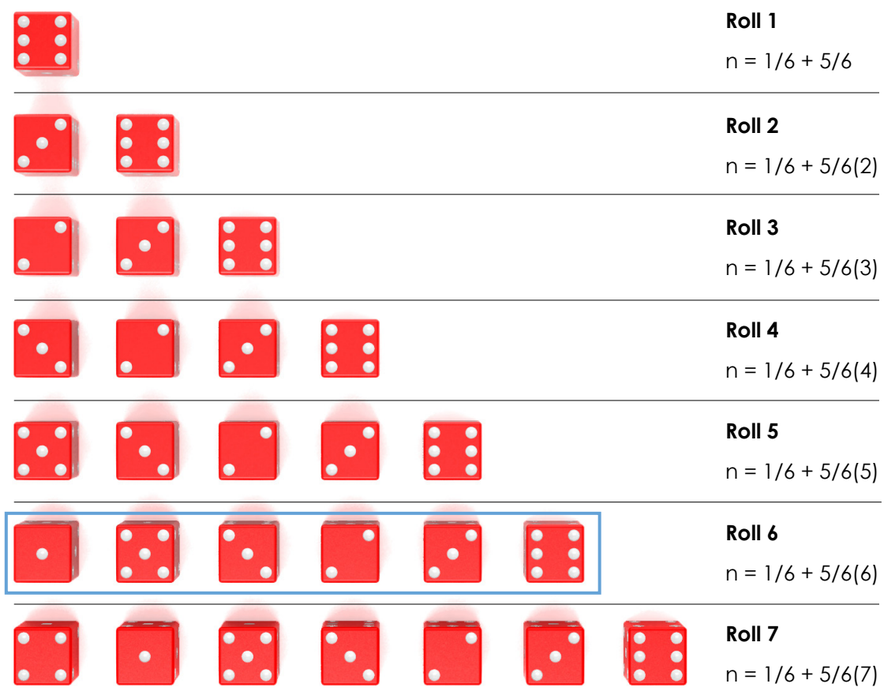

Fig 5: Probability of a six being rolled after previously not getting a six

Getting A Six:

By looking at a sample sequence of dice rolls, we can start to calculate the odds of a six coming up on any particular roll.

It is important to remember that on any individual roll, the odds of rolling a 6 is consistently 1/6.

We can start by look at the odds of continuing a streak of not rolling a six.

What Are The Odds:

Using the equation $x = (5/6)^n$; where n is the number of rolls, we can find the odds of continually not getting a six for each successive roll. We find that after 9.8 (round up to 10) rolls, the odds of getting a six is equal to continuing a non-six streak.

We can also use the similar equation of $y = (1/6)*(5/6)^n$ to find the probability of rolling a six on any particular roll.

Same Probability For Each Roll:

The important part of this puzzle is that the odds of getting a six on each individual roll does not change, and is always 1/6.

After a six is not rolled, then the sequence restarts. Instead of looking at the probability of getting a streak, we need to find the average number of rolls to get a six.

Solution:

We can use an equation that tallies the probabilities of getting a six plus the previous non-six rolls:

$n = (1/6) + (5/6)(n+1)$

Solving this equation, we find that n = 6.

On average it takes 6 rolls to get a six.